

PROFESSOR'S NAME	Prof. Abhijit Das, Prof. Aritra Hazra
DEPARTMENT	Computer Science & Engineering
INSTITUTE	IIT Kharagpur
COURSE OUTLINE	<p>This is a very basic and foundational course needed by all the areas of Computer Science. Computer Science deals with algorithms, and the study of algorithms involves the following major issues.</p> <ol style="list-style-type: none"> 1) Designing algorithms 2) Verifying the correctness of algorithms 3) Analyzing the performance of algorithms <p>The basic skills needed to handle algorithms therefore include logical reasoning, furnishing proofs, and ability to count. This course concentrates mostly on these topics. The issues of countability and uncountability are covered too in order to promote the basic understanding that computers cannot solve all problems. Finally, because of their extensive applications in cryptography and coding theory, the course concludes with a brief study of discrete algebraic structures.</p> <p>Course outline</p> <p>Introduction - Discrete versus continuous mathematics. Relevance to Computer Science.</p> <p>Basic Counting - Sum and product rules; Permutations and combinations with and without repetition; Binomial and multinomial theorems; Catalan Numbers.</p> <p>Propositional Logic - Encoding, reasoning and deductions; Truth tables, satisfiability and validity.</p> <p>Predicate Logic - Predicates, quantifiers and interpretations; Logical deduction, Rules and proofs.</p> <p>Logic Applications in program verification. Proof Techniques - Direct proofs, proof by contradiction and contraposition, proof by cases, cycle of proofs; Mathematical Induction: Weak and strong forms, well-ordering principle; Recursive Constructions; Loop Invariance; Properties of Integers: Divisibility, primes, GCDs, factorizations; Pigeon-hole principle and its applications;</p>

Sets, Functions, and Relations: Sets- Definition and properties, counting using sets, Cartesian products, subsets and power sets; Relations- Definition and properties, equivalence relations and partitions, partial orders, lattices; Functions- Definition and properties (injective, surjective, bijective), composite and inverse functions.

Set Sizes: Finite and infinite sets, countable and uncountable sets; Cantor's diagonal argument and the power set theorem; Cantor-Schroeder-Bernstein theorem; Existence of unsolvable computational problems.

Generating Function: Solving counting problems; Working with infinite series; Partition of integers; Prefix sums and convolution; Solving recurrence relations using generating functions; Catalan numbers.

Recurrence Relations: Formulation and examples: Programming and counting problems, symmetry and fractals; Linear recurrences: Characteristic equations, homogeneous and particular solutions; Divide-and-conquer recurrences and the master theorem.

Discrete Algebraic Structures - Rings and fields: Definitions and properties, homomorphisms and isomorphisms, subrings and ideals, units; Groups: Definitions and properties, homomorphisms and Isomorphisms, cyclic groups, subgroups and cosets; Applications in cryptography and coding theory.