



A Basic Course in Number Theory

SWAYAM Prabha Course Code - S16

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Course Outline	This course intends to develop the basics of number theory touching upon many essential points such as the prime number theorem, quadratic reciprocity laws, Gauss' theorem on the classification of binary quadratic forms, Brahmagupta-Pell equations, to quote a few. This course will enable a student to learn more advanced topics in number theory.

COURSE DETAILS

S. No	Module ID/ Lecture ID	Lecture Title/Topic
1	L1	Integers
2	L2	Divisibility and primes
3	L3	Infinitude of primes
4	L4	Division algorithm and the GCD
5	L5	Computing the GCD and Euclid's lemma
6	L6	Fundamental theorem of arithmetic
7	L7	Stories around primes
8	L8	Winding up on 'Primes' and introducing 'Congruences'
9	L9	Basic results in congruences
10	L10	Residue classes modulo n
11	L11	Arithmetic modulo n , theory and examples
12	L12	Arithmetic modulo n , more examples

13	L13	Solving linear polynomials modulo n - I
14	L14	Solving linear polynomials modulo n - II
15	L15	Solving linear polynomials modulo n - III
16	L16	Solving linear polynomials modulo n - IV
17	L17	Chinese remainder theorem, the initial cases
18	L18	Chinese remainder theorem, the general case and examples
19	L19	Chinese remainder theorem, more examples
20	L20	Using the CRT, square roots of 1 in $\mathbb{Z}/n\mathbb{Z}$
21	L21	Wilson's theorem
22	L22	Roots of polynomials over $\mathbb{Z}/p\mathbb{Z}$
23	L23	Euler ϕ -function - I
24	L24	Euler ϕ -function - II
25	L25	Primitive roots - I
26	L26	Primitive roots - II
27	L27	Primitive roots - III
28	L28	Primitive roots - IV
29	L29	Structure of U_n - I
30	L30	Structure of U_n - II
31	L31	Quadratic residues
32	L32	The Legendre symbol
33	L33	Quadratic reciprocity law - I
34	L34	Quadratic reciprocity law - II
35	L35	Quadratic reciprocity law - III

36	L36	Quadratic reciprocity law - IV
37	L37	The Jacobi symbol
38	L38	Binary quadratic forms
39	L39	Equivalence of binary quadratic forms
40	L40	Discriminant of a binary quadratic form
41	L41	Reduction theory of integral binary quadratic forms
42	L42	Reduced forms up to equivalence - I
43	L43	Reduced forms up to equivalence - II
44	L44	Reduced forms up to equivalence - III
45	L45	Sums of squares - I
46	L46	Sums of squares - II
47	L47	Sums of squares - III
48	L48	Beyond sums of squares - I
49	L49	Beyond sums of squares - II
50	L50	Continued fractions - basic results
51	L51	Dirichlet's approximation theorem
52	L52	Good rational approximations
53	L53	Continued fraction expansion for real numbers - I
54	L54	Continued fraction expansion for real numbers - II
55	L55	Convergents give better approximations
56	L56	Convergents are the best approximations - I
57	L57	Convergents are the best approximations - II
58	L58	Quadratic irrationals as continued fractions

59	L59	Some basics of algebraic number theory
60	L60	Units in quadratic fields: the imaginary case
61	L61	Units in quadratic fields: the real case
62	L62	Brahmagupta-Pell equations
63	L63	Tying some loose ends